LECTURES ON REAL OPTIONS: PART I — BASIC CONCEPTS

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Introduction

- Main Idea: Investment decision can be treated as the exercising of an option.
  - Firm has option to invest.
  - Need not exercise the option now — can wait for more information.
  - If investment is irreversible (sunk cost), there is an opportunity cost of investing now rather than waiting.
  - Opportunity cost (value of option) can be very large.
  - The greater the uncertainty, the greater the value of the firm's options to invest, and the greater the incentive to keep these options open.
- Note that value of a firm is value of its capital in place plus the value of its growth options.

Introduction (Continued)

- Any decision involving sunk costs can be viewed this way:
  - Opening a copper mine.
  - Closing a copper mine.
  - Building an oil tanker.
  - Mothballing an oil tanker.
  - Reactivating a mothballed tanker.
  - Scrapping a tanker.
  - Installing scrubbers on coal-burning power plant.
  - Signing a long-term fuel contract.
  - Undertaking an R&D program.

Introduction (Continued)

- Why look at investment decisions this way? What's wrong with the standard NPV rule?
  - With uncertainty and irreversibility, NPV rule is often wrong — very wrong. Option theory gives better answers.
  - Can value important "real" options, such as value of land, offshore oil reserves, or patent that provides an option to invest.
  - Can determine value of flexibility. For example:
    - Flexibility from delaying electric power plant construction.
    - Flexibility from installing small turbine units instead of building a large coal-fired plant.
    - Flexibility from buying tradeable emission allowances instead of installing scrubbers.
    - Value of more flexible contract provisions.

Introduction (Continued)

- Option theory emphasizes uncertainty and treats it correctly. (NPV rule often doesn't.) Helps to focus attention on nature of uncertainty and its implications.
  - Managers ask: "What will happen (to oil prices, to electricity demand, to interest rates,...)?" Usually, this is the wrong question. The right question is: "What could happen (to oil prices, to...), and what would it imply?"
  - Managers often underestimate or ignore the extent of uncertainty and its implications.
  - Option theory forces managers to address uncertainty.

Planned Sunk-Cost Investment

- Traditional NPV and its limitations.
- Logic of option-theoretic approach.
- Some simple two-period examples.
  - Investing in a widget factory.
  - Investing in a power plant: scale vs. flexibility.
- Projects as perpetual call options. Some basic results and their interpretation.
- Pros and cons of using option-theoretic approach.
- Example: Investments in oil reserves
  - Undeveloped oil reserves as call options.
  - Modelling the price of oil.
  - Basic results.
- Other examples and applications.

Simple NPV Criterion for Project Evaluation

- Net Present Value (NPV) $=$ Present value of inflows – present value of outflows.
- Invest if NPV > 0.
- For example:

$$\text{NPV} = -I_0 - \frac{I_1}{1+r_1} - \frac{I_2}{(1+r_2)^2} + \frac{\text{NCF}_3}{(1+r_3)^3} + \ldots + \frac{\text{NCF}_{10}}{(1+r_{10})^{10}}$$

where:

  - I_t is expected investment expenditure in year t.
  - NCF_t is expected net cash flow from project in year t.
  - r_t is discount rate in year t.

- For the time being, we will keep the discount rate constant for simplicity.

Limitations of Discounted Cash Flow Analysis

- Assumes fixed scenario for outlays and operations. Ignores "option value."
- Examples:
  - Option to delay project
  - Option to stop before completion
  - Option to abandon after completion
  - Option to temporarily stop producing
- How important are these options? Often very important.

Example: NPV With Simple Option

- Project "X" — Not clear it can be a commercial success.
- Two phases:
  - *Phase 1* (Pilot production and test marketing) — Takes 1 year, costs \$125,000.
  - *Phase 2* (Implementation) — Do this only if Phase 1 indicates success. Build \$1 million plant which generates after-tax cash flows of \$250,000 per year forever.
  - Risk: Only a 50 percent chance that Phase 1 will be successful.
- Standard Approach: Risky project; use 25% discount rate, applied to expected values:

$$\text{NPV} = -125 - \frac{500}{1.25} + \sum_{t=2}^{\infty} \frac{125}{(1.25)^t} = -125$$

- Project seems uneconomical. What's wrong?

NPV With Simple Option (Continued)

- Components of project have very different risk characteristics, and *should not be combined*. Phase 1 will resolve most of the risk. If Phase 1 fails, *there is no risk* — project is certain to be worthless.

$$\begin{array}{lll} & \text{Success} \quad \rightarrow & \text{NPV} = -1000 + \sum_{t=1}^{\infty} \frac{250}{(1.1)^t} = 1500 \\ \frac{1}{2} \nearrow & & \\ \frac{1}{2} \searrow & & \\ & \text{Failure} \quad \rightarrow & \text{NPV} = 0 \end{array}$$

- Project has *expected payoff* of $.5(1500) + .5(0) = \$750$, after 1 year and investment of \$125. Using a 30 percent discount rate:

$$\text{NPV} = -125 + \frac{750}{1.3} = 452$$

- Now project looks worthwhile. Point: Be careful when "options" — contingent decisions — are involved.

Another Example with Option

- Consider building a widget factory that will produce one widget per year forever. Price of a widget now is \$100, but next year it will go up or down by 50%, and then remain fixed:

$$
\begin{array}{ccccccc}
\underline{t=0} & & \underline{t=1} & & \underline{t=2} & & \cdots \\
 & & P_1 = \$150 & \longrightarrow & P_2 = \$150 & \longrightarrow & \\
 & \tfrac{1}{2}\nearrow & & & & & \\
P_0 = \$100 & & & & & & \\
 & \tfrac{1}{2}\searrow & & & & & \\
 & & P_1 = \$50 & \longrightarrow & P_2 = \$50 & \longrightarrow &
\end{array}
$$

- Cost of factory is \$800, and it only takes a week to build. Is this a good investment? Should we invest now, or wait one year and see whether the price goes up or down?

Another Example with Option (Continued)

- Suppose we invest now.

$$\text{NPV} = -800 + \sum_{t=0}^{\infty} \frac{100}{(1.1)^t} = -800 + 1,100 = \$300$$

- So NPV rule says we should invest now.
- But suppose we wait one year and then invest only if the price goes up:

$$\text{NPV} = (.5)\left[-\frac{800}{1.1} + \sum_{t=1}^{\infty} \frac{150}{(1.1)^t}\right] = \frac{425}{1.1} = \$386$$

- Clearly waiting is better than investing now.
- Value of being able to wait is $\$386 - \$300 = \$86$.

Another Example (Continued)

- Another way to value flexibility: How high an investment cost I would we accept to have a flexible investment opportunity rather than a "now or never" one?
- Answer: Find $\bar{I}$ that makes the NPV of the project when we wait equal to the NPV when $I = \$800$ and we invest now, i.e., equal to \$300. Substituting $\bar{I}$ for the 800 and \$300 for the \$386 in equation for NPV above:

$$\text{NPV} = (.5)\left[\frac{-\bar{I}}{1.1} + \sum_{t=1}^{\infty} \frac{150}{(1.1)^t}\right] = \$300$$

  Solving for $\bar{I}$ yields $\bar{I} = \$990$.
- So opportunity to build factory now *and only now* at cost of \$800 has same value as opportunity to build the factory now *or next year* at cost of \$990.

Analogy to Financial Options

- Let's solve this simple problem again, but this time using option pricing.
- Next year if the price rises to \$150, we exercise our option by paying \$800 and receive an asset which will be worth

$$V_1 = \$1,650 = \sum_{t=0}^{\infty} \frac{150}{(1.1)^t}$$

- If the price falls to \$50, this asset will be worth only \$550, and so we will not exercise the option.

  - Let F_0 = value today of investment opportunity.

  - Let F_1 = its value next year

Analogy to Financial Options (continued)

- If the price rises to \$150, then

$$F_1 = \sum_{t=0}^{\infty} \frac{150}{(1.1)^t} - 800 = \$850$$

- If the price falls to \$50, the option to invest will go unexercised, so that $F_1 = 0$. Thus we know all possible values for F_1. The problem is to find F_0, the value of the option today.

- To solve this problem, create a portfolio that has two components: the investment opportunity itself, and a certain number of widgets. Pick this number of widgets so that the portfolio is risk-free.

Analogy to Financial Options (continued)

- Consider a portfolio in which one holds the investment opportunity, and sells short n widgets.
- The value of this portfolio today is

$$\phi_0 = F_0 - nP_0 = F_0 - 100n.$$

- Value next year, $\phi_1 = F_1 - nP_1$, depends on P_1.
    - If $P_1 = 150$ so that $F_1 = 850$, $\phi_1 = 850 - 150n$.
    - If $P_1 = 50$ so that $F_1 = 0$, $\phi_1 = -50n$.

Analogy to Financial Options (continued)

- Now, choose n so that the portfolio is risk-free; i.e., so that ϕ_1 is independent of what happens to price. To do this, just set:

$$850 - 150n = -50n,$$

or $\underline{n = 8.5}$. With n chosen this way, $\phi_1 = -425$, whether the price goes up or down.

- We now calculate the return from holding this portfolio. That return is the capital gain, $\phi_1 - \phi_0$, minus any payments that must be made to hold the short position.

- Since the expected rate of capital gain on a widget is zero (the expected price next year is \$100, the same as this year's price), no rational investor would hold a long position unless he or she could expect to earn at least 10 percent.

Analogy to Financial Options (continued)

- Hence selling widgets short will require a payment of $.1P_0 = \$10$ per widget per year. Our portfolio has a short position of 8.5 widgets, so it will have to pay out a total of \$85. The return from holding this portfolio over the year is thus:

$$\begin{aligned}\phi_1 - \phi_0 - 85 &= \phi_1 - (F_0 - nP_0) - 85 \\ &= -425 - F_0 + 850 - 85 = 340 - F_0.\end{aligned}$$

- This return is risk-free, so it must equal the risk-free rate, 10 percent, times the initial portfolio value, $\phi_0 = F_0 - nP_0$:

$$340 - F_0 = .1(F_0 - 850).$$

- Thus $\underline{F_0 = \$386}$. This is the value of the opportunity to build the factory now <u>or</u> next year.

Changes in the Initial Price

- Again fix cost of investment, I, at \$800, but vary initial price, P_0. Whatever P_0 is, $P_1 = 1.5P_0$ or $P_1 = 0.5P_0$, with equal probability.

$$
\begin{array}{cccccc}
\underline{t=0} & & \underline{t=1} & & \underline{t=2} & \cdots \\
 & & P_1 = 1.5P_0 & \rightarrow & P_2 = 1.5P & \rightarrow \\
 & \frac{1}{2} \nearrow & & & & \\
P_0 & & & & & \\
 & \frac{1}{2} \searrow & & & & \\
 & & P_1 = 0.5P_0 & \rightarrow & P_2 = 0.5P_0 & \rightarrow
\end{array}
$$

- To value option, set up risk-free portfolio as before. Value of portfolio today is

$$\phi_0 = F_0 - nP_0$$

Value of a widget factory next year is

$$V_1 = \sum_{t=0}^{\infty} P_1/(1.1)^t = 11P_1$$

Changes in the Initial Price (continued)

- We only invest if V_1 exceeds \$800, so value of option next year is

$$F_1 = \max[0, 11P_1 - 800]$$

Then value of portfolio next year if price goes up is

$$\phi_1 = 16.5P_0 - 800 - 1.5nP_0$$

Value if price goes down is

$$\phi_1 = -0.5nP_0$$

- Equating these two ϕ_1's gives value of n that makes portfolio risk free:

$$n = 16.5 - 800/P_0$$

With n chosen this way, $\phi_1 = -8.25P_0 + 400$ whether price goes up or down.

Changes in the Initial Price (continued)

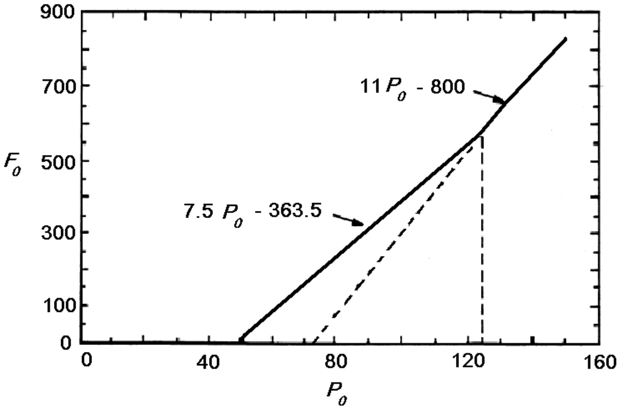
- Calculate return on portfolio, remembering that short position requires payment of $0.1nP_0 = 1.65P_0 - 80$. The return is $6.60P_0 - F_0 - 320$. Since the return is risk free, it must equal $.1\phi_0 = .1F_0 - 1.65P_0 + 80$. Solving for F_0 gives value of option:

$$F_0 = 7.5P_0 - 363.5$$

- We calculated value of option assuming we would only invest if price goes up next year. But if P_0 is low enough, we would never invest, and if P_0 is high enough, it may be better to invest now.
- Below what price would we never invest? From equation for F_0, we see $F_0 = 0$ when $7.5P_0 = 363.5$, or $P_0 = \$48.50$
- For what values of P_0 should we invest now rather than wait? Invest now if current value of factory, V_0, exceeds *total* cost, $\$800 + F_0$. This is the case if $P_0 > \$124.50$.

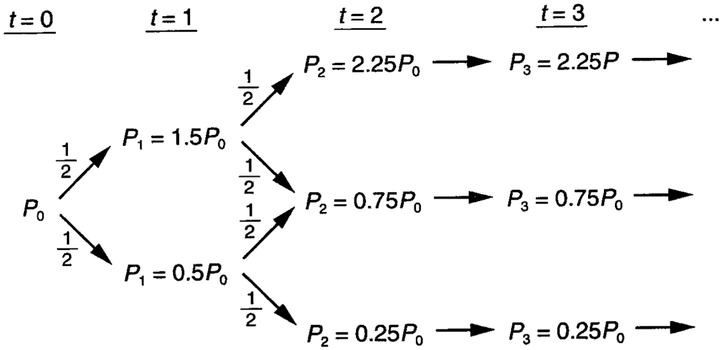
Changes in the Initial Price (continued)

- The figure below shows the value of option to invest as a function of initial price.



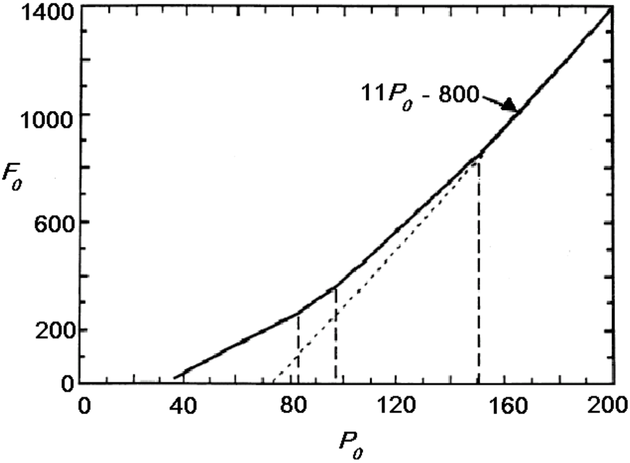
Extension to Three Periods

- We assumed there is no uncertainty over price after the first year. Suppose price can again go up or down by 50% for one more period.



Extension to Three Periods (continued)

- We can calculate option value in same way. Option value shown in figure.



Scale Versus Flexibility

- A utility faces a constant demand growth of 100 Megawatts (MW) per year. It must add to capacity, but how? It has two alternatives:
- Can build a 200 MW coal fired plant (enough for two years' additional demand) at a capital cost of $180 million (Plant A), or a 100 MW oil fired plant at cost of $100 million (Plant B).
- At current coal and oil prices, cost of operating Plant A is $19 million per year for each 100 MW, and cost of Plant B is $20 million per year.
- Discount rate is 10 percent/year, and each plant lasts forever. So if fuel prices remain constant, Plant A is the preferred choice.

Scale Versus Flexibility (continued)

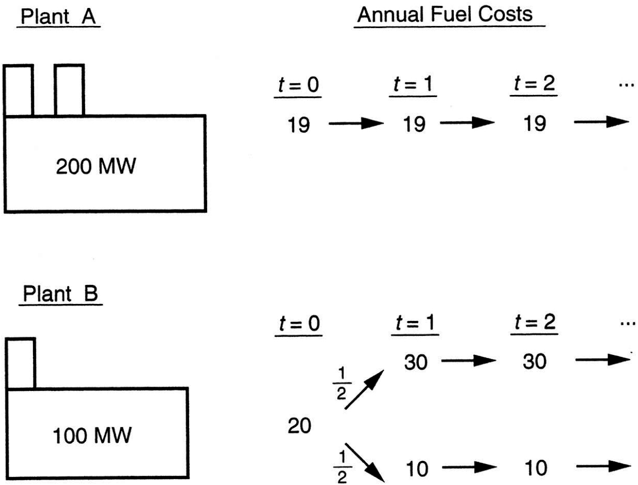


Figure: Choosing Among Electric Power Plants

Scale Versus Flexibility (continued)

- Fuel prices are unlikely to remain constant. Suppose price of coal will remain fixed, but price of oil will either rise or fall next year, with equal probability, and then remain constant. If it rises, operating cost for Plant B will rise to \$30 million/year, but if it falls, operating cost will fall to \$10 million/year.

- Choice is now more complicated. Plant A's capital and operating costs are lower, but Plant B affords more flexibility. If the price of oil falls, utility will not be stuck with the extra 100 MW of coal burning capacity in the second year.

Scale Versus Flexibility (continued)

- Suppose we commit the full 200 MW to either coal or oil:
- With coal, present value of cost is:

$$\mathrm{PV}_A = 180 + \sum_{t=0}^{\infty} \frac{19}{(1.1)^t} + \sum_{t=1}^{\infty} \frac{19}{(1.1)^t} = \$579$$

  Note that 180 is capital cost for the full 200 MW, and 19 is the annual operating cost for each 100 MW, the first of which begins now and the second next year.

- With oil, expected operating cost is \$20 million/year, so present value of cost is:

$$\mathrm{PV}_B = 100 + \frac{100}{1.1} + \sum_{t=0}^{\infty} \frac{20}{(1.1)^t} + \sum_{t=1}^{\infty} \frac{20}{(1.1)^t} = \$611$$

- Thus it seems that Plant A (coal) is best.

Scale Versus Flexibility (continued)

- This ignores flexibility of smaller oil fired plant. Suppose we install 100 MW oil plant now, but then if oil price goes up, install 200 MW of coal fired capacity, rather than another oil plant. This gives total of 300 MW, so to make comparison, net out PV of cost of additional 100 MW, which is utilized starting in two years:

$$\mathrm{PV}_F = \quad 100 + \sum_{t=0}^{\infty} \frac{20}{(1.1)^t} + \tfrac{1}{2}\left[\frac{100}{1.1} + \sum_{t=1}^{\infty}\frac{10}{(1.1)^t}\right]$$

$$+\tfrac{1}{2}\left[\frac{180}{1.1} - \frac{90}{(1.1)^2} + \sum_{t=1}^{\infty}\frac{19}{(1.1)^t}\right] = \$555\,.$$

First term in brackets is PV of costs for second 100 MW oil plant (built if oil price goes down). Second term in brackets is PV of costs of *first 100 MW* of a 200 MW coal plant.

Scale Versus Flexibility (continued)

- This present value is $555 million, so building the smaller oil-fired plant and retaining flexibility is best.
- To value this flexibility, ask how much lower capital cost of Plant A would have to be to make it the preferred choice.
- Let I_A be capital cost of Plant A. So PV of costs of building and running A is:

$$I_A + \sum_{t=0}^{\infty} \frac{19}{(1.1)^t} + \sum_{t=1}^{\infty} \frac{19}{(1.1)^t} = I_A + 399\,.$$

The PV of cost of providing the 200 MW of power by installing Plant B now and then next year installing either Plant A or B (depending on price of oil) is:

Scale Versus Flexibility (continued)

$$
\begin{aligned}
100 \quad &+ \quad \sum_{t=0}^{\infty} \frac{20}{(1.1)^t} + \tfrac{1}{2}\left[\frac{100}{1.1} + \sum_{t=1}^{\infty} \frac{10}{(1.1)^t}\right] \\
&+ \quad \tfrac{1}{2}\left[\frac{I_A}{1.1} - \frac{.5 I_A}{(1.1)^2} + \sum_{t=1}^{\infty} \frac{19}{(1.1)^t}\right] \\
&= \quad 320 + \tfrac{1}{2}(90.9 + 100) + \tfrac{1}{2}(.496 I_A + 190) \\
&= \quad 510.5 + .248 I_A \, .
\end{aligned}
$$

- To find capital cost that makes us indifferent between these choices, equate these PVs and solve for I_A:

$$I_A + 399 = 510.5 + .248 I_A \, ,$$

or, $I_A^* = \$148.3$ million.

Scale Versus Flexibility (continued)

- Conclusion: Scale economies must be large (so that the cost of 200 MW coal plant was less than 75 percent of the cost of two 100 MW oil plants) to make giving up the flexibility of the smaller plant economical.
- Here, only uncertainty was over fuel prices. Could also find value of flexibility when there is uncertainty over:
    - Demand growth.
    - Future capital costs (e.g., because of uncertainty over future environmental regulations).
    - Interest rates.

Cost and Revenue Uncertainty

- You must decide whether to make initial \$15 million investment in R&D. Later, if you continue, more money will be invested in a production facility.
- Three possibilities for cost of production, each with probability $\frac{1}{3}$:
  - Low (\$30 million)
  - Medium (\$60 million)
  - High (\$120 million)
- Two possibilities for revenue (each with probability $\frac{1}{2}$):
  - Low (\$50 million)
  - High (\$110 million)
- Should you invest the \$15 million?

$$\begin{aligned} \text{NPV} &= -\$15 + \tfrac{1}{2}(50) + \tfrac{1}{2}(110) - \tfrac{1}{3}(30) - \\ &\qquad \tfrac{1}{3}(60) - \tfrac{1}{3}(120) \\ &= -\$5 \text{ million} \end{aligned}$$

- NPV is negative, so it seems you should not invest.

Cost and Revenue Uncertainty (continued)

- But suppose \$15 million R&D reveals cost of production. Assume (for now) expected revenue of \$80 million. Hence proceed with production only if cost is low or medium:
- Low Cost: $\Pi = 80 - 30 = \$50$ million
- Medium Cost: $\Pi = 80 - 60 = \$20$ million

$$\begin{aligned} \mathrm{NPV}_1 &= -\$15 + \tfrac{1}{3}(0) + \tfrac{1}{3}(50) + \tfrac{1}{3}(20) \\ &= \$8.33 \text{ million} \end{aligned}$$

So investment in R&D is justified — it creates an option.

Cost and Revenue Uncertainty (continued)

- Suppose you can postpone production until you learn whether revenue is low or high. Then better to wait: If revenue is low, don't produce unless cost is low. Now:

$$\begin{aligned} \mathrm{NPV}_2 &= -\$15 + \tfrac{1}{3}(0) + \tfrac{1}{3}\left[\tfrac{1}{2}(50) + \tfrac{1}{2}(110) - 30\right] \\ &\qquad + (\tfrac{1}{3})(\tfrac{1}{2})(110 - 60) \\ &= \$10 \text{ million} \end{aligned}$$

Discount Rates for Risky R&D

- Suppose you want to value the following sorts of projects:
  - Development and testing of a new drug.
  - Development of a data compression method which may or may not work, and may or may not have a market.
  - Development of a low power microprocessor for laptops and PDAs (e.g., Transmeta).
  - Early-stage oil and gas exploration in a new and uncharted area.
- In each case, high risk of failure. Unlikely you will end up with a commercially successful product.
- If you do succeed, high payoff.
- Risk of success or failure is diversifiable. No systematic risk until sales of commercial product (should you succeed).
- What discount rate should you use when deciding whether to begin the project? What is the correct "beta" for the project?

Leverage Effect

- Project is like a *compound option*: each stage, if successful, gives you an option to do the next stage. Creates leverage.
- Suppose risk-free rate is $r_f = 5\%$ and market risk premium is $r_m - r_f = 5\%$. Suppose that *if project is successful*, resulting net revenue has $\beta = 1$, so discount rate is $r_{NR} = r_f + 1(r_m - r_f) = 10\%$.
- Assume R&D risk is completely diversifiable, so discount rate for cost of R&D is $r_f = 5\%$.
- NPV of project is $PV_{NR} - PV_C$, so $PV_{NR} = PV_C + NPV$

Leverage Effect (continued)

- Thus expected return on PV_{NR} must equal weighted average return on PV_C and NPV:

$$r_{NR} PV_{NR} = r_C PV_C + r^* NPV$$

$$r^* = [r_{NR} PV_{NR} - r_C PV_C] / NPV$$

- We can rewrite this as:

$$\boxed{r^* = r_{NR} + (r_{NR} - r_C)\frac{PV_C}{NPV}}$$

- In most cases, $r_{NR} > r_C$, so that $r^* > r_{NR}$. If $PV_C >> NPV$, then $r^* >> r_{NR}$.

Leverage Effect (continued)

- What is the equivalent β?

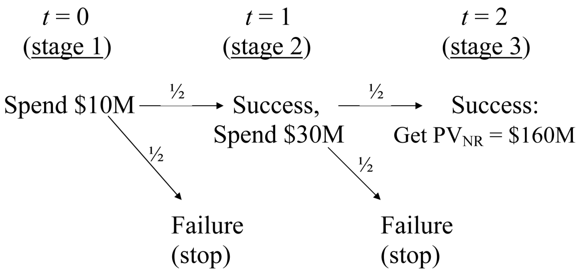
$$r^* = r_f + \beta(r_m - r_f)$$

$$\beta^* = (r^* - r_f)/(r_m - r_f)$$

- Suppose $\beta_{NR} = 1$ so that $r_{NR} = r_m$. Then $r^* > r_m$, and $\beta^* > 1$.
- This is the leverage effect. Not that it has *nothing to do with any adjustment for the riskiness of the R&D*.

A Simple Example

- Three-stage project.
  - At $t = 0$ (Stage 1), spend \$10 million on R&D. Probability of success $= \frac{1}{2}$. If successful:
  - At $t = 1$, spend \$30 million on next stage of R&D. Probability of success $= \frac{1}{2}$. If successful:
  - At $t = 2$, you have a commercial product that generates net revenues with $\beta_{NR} = 1$ and PV_{NR} =\$160 million.



A Simple Example (continued)

- Stage 1

$$PV_{NR} = (\tfrac{1}{2})^2 \frac{160}{(1+r_{NR})^2} = \frac{40}{(1.1)^2} = \$33.1M$$

$$PV_c = 10 + (\tfrac{1}{2}) \frac{30}{1.05} = \$24.3M$$

$$NPV = 33.1 - 24.3 = \$8.8M$$

- What is cost of capital at Stage 1?

$$r_1^* = r_{NR} + (r_{NR} - r_C) \frac{PV_C}{NPV}$$

$$= .10 + (.05) \frac{24.3}{8.8} = 23.8\%$$

- What is equivalent β for Stage 1?

$$r^* = r_f + \beta(r_m - r_f) = .05 + .05\beta$$

$$\beta_1^* = (.238 - .05)/.05 = 3.76$$

A Simple Example (continued)

- Stage 2 (assuming Stage 1 is successful)

$$PV_{NR} = \left(\tfrac{1}{2}\right)\frac{160}{1.1} = \$72.7M$$

$$PV_C = \$30M$$

$$NPV = 72.7 - 30 = \$42.7M$$

- What is the cost of capital at Stage 2?

$$r_2^* = .10 + (.05)\frac{30}{42.7} = 13.5\%$$

- What is β for Stage 2?

$$\beta_2^* = (.135 - .05)/.05 = 1.70$$

A Simple Example (continued)

- SUMMARY

	r^*	β^*
Stage 1 R&D:	23.8%	3.76
Stage 2 R&D:	13.5%	1.70
Production and Sales:	10%	1.00

Questions

- Here are some recent estimates of equity betas:

Merck:	$\beta = 0.4$
Onxx Pharmaceuticals:	$\beta = 1.90$
Amazon.com:	$\beta = 2.55$
eBay:	$\beta = 2.62$

- Merck has been developing a large number of new molecules. Why is its beta so low?
- Onxx is developing a viral anti-cancer agent and doing little else. Is this conisitent with a beta of 1.90?
- Amazon and eBay are relatively mature companies. Why are their betas around 2.5?
- In our simple example, NPV at Stage 1 was positive but small. Would this be representative of a typical startup project?